This full text paper was peer-reviewed at the direction of IEEE Instrumentation and Measurement Society prior to the acceptance and publication.

A Robust Quaternion based Kalman Filter Using a Gradient Descent Algorithm for Orientation Measurement

1st Lukas Wöhle
Electrical Engineering and Applied Sciences
Westphalian University of Applied Sciences
Gelsenkirchen, Germany
lukas.woehle@w-hs.de

2nd Marion Gebhard
Electrical Engineering and Applied Sciences
Westphalian University of Applied Sciences
Gelsenkirchen, Germany
marion.gebhard@w-hs.de

Abstract*—This paper presents a robust quaternion based filter approach to estimate orientation from Magnetic Angular Rate Gravity (MARG)-sensors. These sensors consist of tri-axis accelerometer and gyroscope as well as a tri-axis magnetometer which allow a complete measurement of orientation relative to the direction of gravity and magnetic field of the earth. The proposed filter uses a gradient descent algorithm (GDA) to compute an orientation from magnetometer and accelerometer data. This calculated orientation is directly used as an input in a Kalman filter framework (KFF) which predicts the orientation estimation from gyroscope data. The embedding of the gradient descent algorithm into the Kalman filter allows the computation of a weighted orientation represented as quaternion. Furthermore, the designed filter can overcome short time magnetic disturbance by switching between MARG and IMU equations inside the gradient descent filter stage (GDFS) and therefore enables a more robust orientation estimation without the need for additional algorithms. Tests show the proposed filter to be superior to a commercially available sensor fusion algorithm related to orientation estimation at slow angular rates. Moreover, the proposed filter is able to maintain good orientation estimation under short term magnetic disturbance.

***Index Terms*—sensor fusion, orientation estimation, magnetic disturbances, gradient descent algorithm, Kalman filter**

I. Introduction

The combination of inertial and magnetic sensors can be used to measure a complete orientation in three-dimensional-space relative to the direction of earth's gravity and magnetic field. These types of sensors, termed MARG-sensors, are completely self-contained and therefore not limited to a specific type of motion nor environment. These characteristics ensure a wide variety of applications in different fields such as aerospace [1], robotics [2], human motion analysis [3] or human-machine interaction [4]. All applications in the previous mentioned fields need a robust, precise and computationally efficient algorithm for orientation estimation to work in environments which are exposed to magnetic disturbance [5], known as hard iron and soft iron effects [6].

This work was funded by the Federal Ministry of Education and Research of Germany.

II. State of the Art sensor fusion

The underlying method of data fusion is key to compute an orientation based on the measured quantities. Many different algorithms exist to fulfil the orientation estimation. The Kalman filter has become the most commonly used one [7], [8] even in commercial applications [9]. However, a fairly new method of orientation estimation has been introduced by Madgwick et al. in 2011 [10]. This method is based on the gradient descent algorithm (GDA) and has gained popularity over the past years. A crucial benefit of this algorithm is its simple and effective implementation of the filter framework on a microcontroller. This filter comes along with only two filter gains to control the fusion of sensor data and is applicable to both IMU and MARG-sensors, provided that the sensors are calibrated. A calibrated magnetometer will sense a unique change of direction in the surrounding magnetic field [11]. This is essential for precise orientation estimation, especially regarding MARG-sensor data fusion. However, in almost every environment the MARG-sensors are exposed to magnetic field disturbance, like soft iron or hard iron effects, resulting in incorrect orientation estimation [12]. While Madgwicks approach is computational efficient but suffers from magnetic disturbance in the heading estimate, recent approaches try to overcome these magnetic disturbances by either extra Hardware, e.g. magnetic sources [13], or software, e.g. through cascaded two step Kalman filters [14]. The first step incorporates gyroscope as well as accelerometer data to estimate orientation without using geomagnetic field data for correction of the yaw angle - thus using IMU-sensors only. The second step incorporates magnetometer data and only contributes to orientation estimation when it is not affected by magnetic disturbance. This enables an orientation estimation to be computed even under magnetic disturbance. Nevertheless, this results in a large model regarding matrix operations and outputs Euler angles rather than quaternions. Another approach incorporates a GDA in a Kalman filter [15] and compensates magnetic disturbances. This filter computes a quaternion from accelerometer data which in addition to

978-1-5386-2222-3/18/$31.00 ©2018 IEEE

the angular velocity data is the input to a Kalman filter. The outputs are pitch and roll angle only. The magnetometer data is used to calculate yaw angles by Kalman filter outputs and ellipse hypothesis compensation. However, this method lacks to calculate a complete quaternion representation and incorporates magnetometer data inside a third calculation.

This paper presents an approach to estimate orientation via a two-stage filter consisting of a GDA stage and a Kalman filter framework (KFF) to overcome short time magnetic disturbance compromising the heading estimate. It is done by switching from MARG- to IMU-type fusion and vice versa inside the gradient descent filter stage (GDFS). This stage employs a quaternion representation and is treated like a measurement inside the Kalman filter. The KFF incorporates the gyroscope model as quaternion and calculates a final orientation, represented as quaternion. The computation of the orientation inside the Kalman filter scales with respect to noise from motion and allows for weighted solutions based on the sensor characteristics. The filter is benchmarked against a commercial filter and its effectiveness under magnetic disturbance.

III. Filter Development

A. Orientation from angular rate

A gyroscope measures the angular rate of the sensor in its body frame and is denoted as ω. This process can be represented as a quaternion (${}^N_B\mathbf{q}$) describing the rate of change of the orientation of the global navigation frame (index N) relative to the body frame (subscript B) [16] and can be written as

$$ {}^N_B\dot{\mathbf{q}}_{\omega,t} = -\frac{1}{2}\begin{pmatrix} 0 \\ {}^B\vec{\omega}_t \end{pmatrix} \bullet \ {}^N_B\mathbf{q}_{t-1} \tag{1} $$

where $\bullet$ denotes the quaternion multiplication and ${}^B\vec{\omega}_t$ is the angular velocity vector of the three-axis arranged

$$ {}^B\vec{\omega}_t = \begin{pmatrix} \omega_{x,t} & \omega_{y,t} & \omega_{z,t} \end{pmatrix}^T . \tag{2} $$

Under known initial conditions the angular velocity may be integrated over time to compute the sensor's orientation relative to the navigation frame. This orientation ${}^N_B\mathbf{q}\omega, t$ can be computed by numerically integrating the quaternion derivate ${}^N_B\dot{\mathbf{q}}_{\omega,t}$ from (1) and given initial conditions

$$ {}^N_B\mathbf{q}_{\omega,t} = \ {}^N_B\mathbf{q}_{t-1} + \ {}^N_B\dot{\mathbf{q}}_{\omega,t}\Delta t \tag{3} $$

B. Orientation from earth magnetic and gravity field

As proposed by Madgwick et al. [10] one can estimate the orientation of a sensor $\mathbf{q}^n_b$ through vector observation. The orientation of a sensor relative to the global navigation frame can be found as a quaternion which transforms a predefined reference field with known direction $\vec{d}^n$ (i.e. gravity or magnetic field) into the measured direction of this field in the sensor frame $\vec{s}^b_b$. There will be an infinite number of possible orientation solutions. However, a quaternion representation requires a single solution which leads to the formulation of an optimisation problem.

$$ \min_{{}^N_B\mathbf{q}\in\mathbb{R}^4} \ f({}^N_B\mathbf{q}, {}^N\vec{d}, {}^B\vec{s}) \tag{4} $$

where

$$ f({}^N_B\mathbf{q}, {}^N\vec{d}, {}^B\vec{s}) = {}^N_B\mathbf{q} \bullet \begin{pmatrix} 0 \\ {}^N\vec{d} \end{pmatrix} \bullet {}^N_B\mathbf{q}^* - \begin{pmatrix} 0 \\ {}^B\vec{s} \end{pmatrix} . \tag{5} $$

Madgwick et al. proposed the GDA as a possible solution to this optimisation problem because of its low implementation and computational costs [10] leading to the general form

$$ {}^N_B\mathbf{q}_{k+1} = {}^N_B\mathbf{q}_k - \mu_t \frac{\nabla f({}^N_B\mathbf{q}_k, {}^N\vec{d}, {}^B\vec{s})}{||\nabla f({}^N_B\mathbf{q}_k, {}^N\vec{d}, {}^B\vec{s})||}, \ k = 0, 1, 2, ...n \tag{6} $$

where μ_t is the step size of the GDA and ∇f denotes the gradient defined as the product of the objective function f and its Jacobian J

$$ \nabla f({}^N_B\mathbf{q}, {}^N\vec{d}, {}^B\vec{s}) = J^T({}^N_B\mathbf{q}, {}^N\vec{d}) f({}^N_B\mathbf{q}, {}^N\vec{d}, {}^B\vec{s}). \tag{7} $$

The equation simplifies if the direction of the reference fields is assumed to only have components in one or two major axes of the global frame. Following Madgwicks convention this leads to the assumption that the gravity field defines the vertical z-axis and the magnetic field only has components in the x- and y-axis due to the inclination of the magnetic field. This breakdown results in two objective functions and their Jacobian for gravity and magnetic field respectively [10]. Depending on the desired application, either attitude and heading for MARG-sensors or just attitude for IMU-sensors, ∇f has to be chosen. Substituting ${}^B\vec{s}$ for normalised accelerometer (${}^B\vec{a}_t$) or magnetometer measurement (${}^B\vec{m}_t$) and ${}^N\vec{d}$ for either normalized gravity or normalised and distortion compensated magnetic field vector (${}^N\vec{b}$) results in a set of equations to form the gradient of function f (∇f)

$$ \nabla f_a = J^T_a({}^N_B\mathbf{q}_{t-1}) f_a({}^N_B\mathbf{q}_{t-1}, {}^B\vec{a}_t) \tag{8} $$

$$ \nabla f_{a,m} = J^T_{a,m}({}^N_B\mathbf{q}_{t-1}, {}^N\vec{b}), f_{a,m}({}^N_B\mathbf{q}_{t-1}, {}^B\vec{a}_t, {}^N\vec{b}, {}^B\vec{m}_t) \tag{9} $$

where the subscript a indicates equations for IMU and a, m for MARG-sensor mode.

The combination of gravity and magnetic field measurements enables a complete orientation to be found. Conventional approaches would require multiple iterations for each new iteration but as Madgwick et al. [10] states it is acceptable to compute one orientation per time sample providing that the convergence rate is equal or greater than the physical rate of change of the device. This is governed by the step size (μ_t). Equation (10) calculates a complete quaternion based on the previous orientation and solution from the GDA per sample time t

$$ {}^N_B\mathbf{q}_{\nabla,t} = {}^N_B\mathbf{q}_{t-1} - \mu_t \frac{\nabla f}{||\nabla f||} \tag{10} $$

where ${}^N_B\mathbf{q}_{\nabla,t}$ is the quaternion computed via the GDA and μ_t can be calculated as follows

$$ \mu_t = \alpha||{}^N_B\dot{\mathbf{q}}_{\omega,t}||\Delta t, \ \alpha > 1. \tag{11} $$

The entire mathematical derivation of the filter can be found in [10].

C. Robust quaternion based Kalman filter

Within this work it is proposed to fuse the quaternion (${}^N_B\mathbf{q}_{\nabla,t}$) calculated on the basis of accelerometer and magnetic field sensor data and the quaternion (${}^N_B\mathbf{q}_{\omega,t}$) calculated on the basis of gyroscope data by using a linear Kalman filter. The computation of only one quaternion from both the magnetometer and accelerometer data reduces the size of the needed model and state vectors. Furthermore (8) and (9) enables the filter to switch between the MARG- and the IMU- mode if the sensor is subject to short time magnetic disturbances. Thereby remaining the effectiveness on implementation of the GDA. The state vector x_{k+1} of the proposed Kalman Filter consists of the quaternion components

$$x_{k+1} = {}^N_B\mathbf{q} = \begin{pmatrix} q_0 & q_1 & q_2 & q_3 \end{pmatrix}^T. \tag{12}$$

Following (1) and (3) for orientation estimation the predicted quaternion is computed through integration of the angular rate measured by the tri-axis gyroscope [16]. The vector differential equation describing the rate of change (1) in matrix form can be rewritten as

$${}^N_B\dot{\mathbf{q}}_{\omega,t} = \Omega({}^B\omega_t){}^N_B\mathbf{q}_t \tag{13}$$

where

$$\Omega({}^B\omega_t) = \begin{pmatrix} 0 & -{}^B\omega_{x,t}^T & -{}^B\omega_{y,t}^T & -{}^B\omega_{z,t}^T \\ {}^B\omega_{x,t} & 0 & {}^B\omega_{z,t} & -{}^B\omega_{y,t} \\ {}^B\omega_{y,t} & -{}^B\omega_{z,t} & 0 & {}^B\omega_{x,t} \\ {}^B\omega_{z,t} & {}^B\omega_{y,t} & -{}^B\omega_{x,t} & 0 \end{pmatrix}. \tag{14}$$

Discretization and added process noise (w_k) to compensate for model errors results in the discrete state transition equation

$$x_{k+1} = \Phi({}^B\omega, \Delta t)x_k + w_k \tag{15}$$

where $\Phi({}^B\omega, \Delta t)$ denotes the state transition matrix and is computed using zero order integration yielding

$$\Phi({}^B\omega, \Delta t) = \left(I_{4\times4} + \frac{1}{2}\Omega({}^B\omega)\Delta t\right). \tag{16}$$

The discrete process noise (w_k) can be modelled according to (1) as the noise affecting the gyroscopes reading as quaternion derivate, where instead of ω white Gaussian measurement noise v_k with covariance matrix Σ_g is being used

$$\Sigma_g = \sigma_g^2 I_{3\times3} \tag{17}$$

where σ_g is the standard deviation of the gyroscope readings. Switching multiplicands and expanding (1) w_k can be written as

$$w_k = -\frac{\Delta t}{2}\begin{pmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & -q_3 & -q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{pmatrix} v_k. \tag{18}$$

According to (18) the process noise covariance matrix Q_k can be computed

$$Q_k = (\frac{\Delta t}{2})^2 \begin{pmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & -q_3 & -q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{pmatrix} \Sigma_g \begin{pmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & -q_3 & -q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{pmatrix}^T. \tag{19}$$

The update step applies the quaternion ${}^N_B\mathbf{q}_{\nabla,t}$ from (10) and adds measurement noise. The discrete process model equation for the update can simply be written as:

$$z_k = {}^N_B\mathbf{q}_{\nabla,k-1} = Hx_k + v_{zk} \tag{20}$$

where ${}^N_B\mathbf{q}_{\nabla,k-1}$ represents the quaternion computed through GDA, H is a 4×4 identity matrix and v_{zk} is noise associated with the estimation of orientation through the GDA based on the sensors measurement noise which is assumed to be white. Noting that the quaternion computed through GDA is based on accelerometer and magnetometer, thus v_{zk} is related to accelerometer and magnetometer measurement noise.

The measurement noise covariance matrix (R_k) takes a major role regarding the fusion process of the filter. Under dynamic motion the accelerometer will be subject to dynamic linear acceleration which influences the measured direction of the gravity vector thus resulting in a wrong computation of orientation inside the GDA. To avoid the effect of a wrong measurement quaternion onto the orientation estimation, a gain factor is introduced to weight the fusing of the measurement quaternion with the quaternion computed via angular velocity. In case of a Kalman filter this fusion gain is implemented in the measurement noise covariance matrix. In phases of high dynamic motion, the covariance matrix should be updated with a high gain. The magnitude of gain scales with respect to the physical rate of change resulting in a decreasing influence of the measurement on the solution. In contrast, the contribution of the measurement in static or slow movements phases should be very high, keeping gyroscope errors at a minimum [17]. This scaling is achieved through multiplication of the measurement covariance matrix with the step size μ_t of the GDA. In phases of high dynamic motion, the step size will rise due to increased angular rates resulting in a solution heavily based on the gyroscope measurement. At rest or slow movements the orientation estimation is mainly composed of the measurement quaternion from GDA. R_k can be written as

$$R_k = \Sigma_{a,m} + I_{4\times4}\mu_t,\ \Sigma_{a,m} = E[v_{zk}v_{zk}^T] \tag{21}$$

where $\Sigma_{a,m}$ represents the covariance matrix and equals noise introduced from both sensors affecting the estimation of a quaternion inside the GDA. It is determined using measurement data and results in a diagonal 4×4 matrix. The GDA converges over time as long as the physical rate of change is smaller than the convergence rate which is governed by the step size μ_t [10]. Therefore, the step size is the main error source during dynamic motion for the quaternion from GDA.

D. Maintaining orientation under magnetic disturbance

It is possible to calculate orientation either through MARG or IMU like filter fusion by switching in between equation (8) and (9) in the update step of the filter. Using both the GDA and the KFF it is possible to detect magnetic disturbance or uncalibrated magnetometer measurements. This is crucial because in distorted fields the quaternion calculated from GDA will not result in the correct orientation.

Within this work the following concept is proposed. The

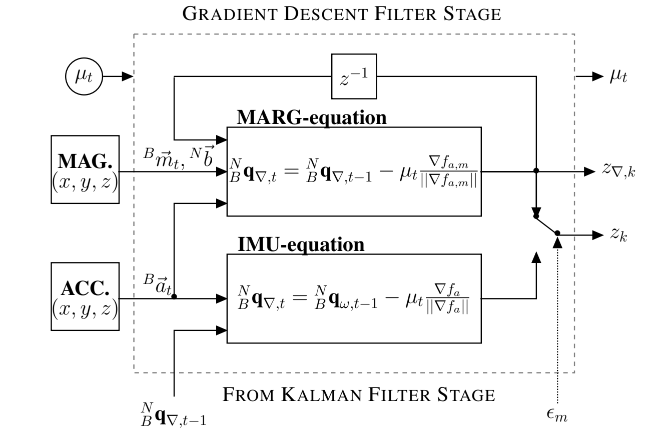

Fig. 1. Block diagram of the gradient descent filter stage (GDFS) with threshold based switching.

calculation of the gradient is separated from the gyroscope readings. The estimated quaternion from magnetometer and accelerometer data in equations (9) and (10) is used as an input to the KFF. The quaternion from the MARG- and IMU-equations are used as measurement input to the KFF. This increases the time needed to converge to a steady state inside the GDFS but allows to detect magnetic disturbance inside the Kalman filter. The block diagram in Fig. 1 shows the concept. The quaternion calculated on basis of acceleration data and magnetic field data represents the input $z_{\nabla,k}$. As the difference (y_k) between the global quaternion from GDA and the quaternion computed using the gyroscope when applied to the filter in the update step is quite large, magnetic disturbances are recognised

$$y_k = z_{\nabla,K} - Hx_k. \tag{22}$$

To avoid the effect of short time disturbed magnetic fields on the quaternion and therefore the estimated yaw angle it is possible to switch the measurement quaternion equation (z_k) to the IMU case of equations (8) through *threshold-based switching*. Therefore $|y_k|$ is computed and tested for large differences against a criteria threshold ϵ_m. This constant is the criteria for magnetic disturbance recognition and is determined through experiments to provide best switching capabilities while subject to magnetic disturbance. Upon switching towards the IMU equations it is required to switch the input of the quaternion from the previous timestep to the quaternion based on the gyroscope readings from the Kalman filter because the quaternion from accelerometer only defines the vertical z-axis. In order to provide a fast convergence of the GDFS and create a larger difference y_k under magnetic disturbance, a gain factor c_b is added to the calculation of (8) and (9) resulting in the following cases

$$z_k = \begin{cases} {}^N_B\mathbf{q}_{\nabla,t} = {}^N_B\mathbf{q}_{\nabla,t-1} - (\mu_t + c_b)\frac{\nabla f_{a,m}}{||\nabla f_{a,m}||}, & |y_k| < \epsilon_m \\ {}^N_B\mathbf{q}_{\nabla,t} = {}^N_B\mathbf{q}_{\omega,t-1} - (\mu_t + c_b)\frac{\nabla f_a}{||\nabla f_a||} \end{cases} . \tag{23}$$

where ${}^N_B\mathbf{q}_{\omega,t-1}$ equals the a priori state vector $\hat{x}_k^-$ and

$$c_b = \begin{cases} |y_k|, & |y_k| < \epsilon_m \\ 0.1 \end{cases} . \tag{24}$$

During magnetic disturbance both equations from (23) need to be calculated to keep the magnetic disturbance tracked in the form of y_k, which relies on the MARG-equation ($z_{\nabla,k}$). If the disturbance vanishes the fusion again applies the MARG-case. However, this depends on the period of the disturbance. The IMU based solution is subject to gyroscope drift and therefore y_k will scale over time making it difficult to get back to the MARG-case without large errors.

The fact that this filter computes a complete quaternion solution introduces one general problem. While an orientation can be expressed as different quaternions the filter needs to calculate similar quaternions to switch from IMU to MARG case by comparing the GDA quaternion and the state vector. When the sensor is calibrated properly y_k will be small. If the sensor is not calibrated, y_k scales up. A complete rotation of the device around a major axis might end up in different quaternions for the GDA quaternion and state vector representing nearly the same orientation. In such cases, $|y_k|$ will become either 2 or 0 because of the norm of the subtraction of two unit quaternions. At this point the orientation is the same but the quaternions differ. This case can be addressed by setting the previous state vector x_k to z_k resolving different quaternions for the same orientation representation.

Finally the step size μ_t can be computed. Equation (11) expects the rate of change of the orientation to compute an adaptive step size. This rate of change can be computed through (1), where ${}^N_B\mathbf{q}_{t-1}$ is formed through (23) which yields that μ_t can be calculated as follows

$$\mu_t = \alpha \left|\left| \left(-\frac{1}{2} \begin{pmatrix} 0 \\ \vec{\omega}_b^n \end{pmatrix} \bullet z_{k-1} \right) \right|\right| \Delta t. \tag{25}$$

A complete diagram of both filter stages and their dependencies is depicted in Fig. 2.

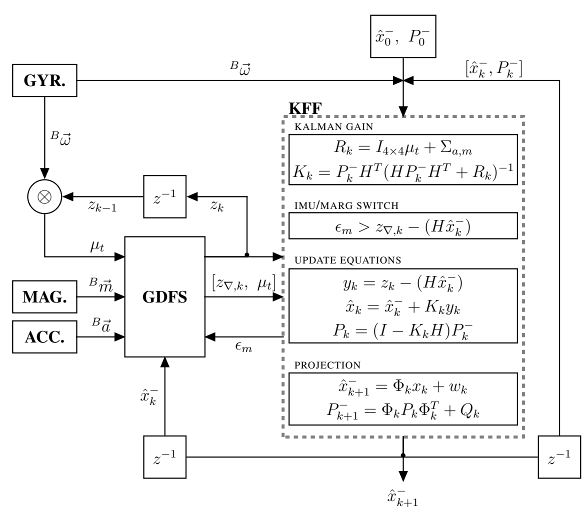

Fig. 2. Diagram of both filter stages, the Kalman filter framework (KFF) and the gradient descent filter stage (GDFS), and their interconnections.

IV. Experimantel Setup and Equipment

The proposed filter was implemented in MATLAB and tested using a FSM-9 module from Hillcrest Labs. The FSM-9 consists of a tri-axis gyroscope, magnetometer and accelerometer and features full 3D motion processing on a 32-bit MPU, including quaternion output[1]. The FSM-9 was programmed to output factory calibrated values sampled at a rate of 125 Hz and directly feed to the proposed filter. The sensor was calibrated to the surrounding magnetic field according to the manufacturers advice.

A robot (*UR5*) was used to rotate the sensor to provide accurate orientation changes that are used as the ground truth to compare the algorithms. The motor encoders of the *UR5* have a resolution of $0.01^\circ s^{-1}$. A second MARG-sensor, the XDK110 Cross-Domain Development Kit from BOSCH, was programmed to output raw values, calibrated according to [11] working as a redundant source of reference to validate correctness of data. A set of 16 rotations around every major axis (x, y, z) of the sensor, where each rotation corresponds to 180° total orientation change in the particular axis, was recorded. Regarding dynamic and static performance of the filter each rotation sequence was recorded with decreasing angular rates, starting at an angular rate of $18^\circ s^{-1}$ on the first rotation and completing at $3^\circ s^{-1}$. Every set was recorded five times for the undistorted case and five times for the magnetic distorted case to investigate behaviour under magnetic disturbance. Artificial magnetic distortions were introduced by bringing an iron bar close to the sensors ($4\ cm$) for less than 35 seconds disturbing the surrounding magnetic field by introducing soft iron effects.

V. Experimantel Results

The performance of orientation estimation is presented as static and dynamic RMS errors (subscript ϵ) of the Euler angles, roll (ϕ), pitch (θ) and yaw (ψ). These are calculated as the difference between estimated orientation and the ground truth obtained from the robot joint angle. The proposed algorithm parameters are as follows:

- Parameters of noise: $\sigma_g^2 = 0.0001$, $\Sigma_{a,m} = I_{4\times4} * 0.001$
- Initial states: State covariance matrix, P_0, was initially set to be the identity Matrix $I_{4\times4}$ and state vector, x_0, was set to unit quaternion $[1\ 0\ 0\ 0]^T$.
- Gradient descent scaling factor $\alpha = 10$ and magnetic disturbance threshold $\epsilon_m = 0.065$.
- At startup, c_b was set to 5 and ϵ_m to 50 for 5 seconds to let the GDA converge faster towards the initial orientation defined by direction of gravity and magnetic field vector.

The estimated angle errors are divided into three groups, static, slow and fast dynamic where static equals no motion, slow dynamic equals angular rates of $3^\circ s^{-1} \le \omega \le 6^\circ s^{-1}$ and fast dynamic is $\omega > 6^\circ s^{-1}$. At static and fast dynamic motions both, the commercial filter and the one proposed here show similar performance regarding motion estimation (Fig. 3, left). In the case of slow dynamic motion the commercially available sensor fusion algorithm of the FSM-9 module

[1]http://hillcrestlabs.com/products/fsm-9/

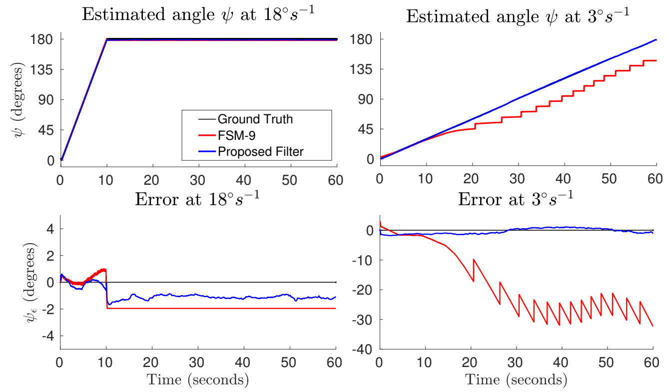

Fig. 3. Typical results for ground truth and estimated angle ψ (top) and corresponding error (bottom) at fast ($18^\circ s^{-1}$) and slow ($3^\circ s^{-1}$) movements.

TABLE I
RMS values for commercial implemented filter (FSM-9) and proposed filter (HyKG) under three different movement constraints - static, slow dynamic and fast dynamic.

	Static		*Dynamic (slow)*		*Dynamic (fast)*	
	FSM-9	HyKG	FSM-9	HyKG	FSM-9	HyKG
ϕ_ϵ	0.55°	0.54°	0.51°	0.71°	0.51°	0.78°
θ_ϵ	0.48°	0.45°	0.41°	0.62°	0.33°	0.41°
ψ_ϵ	1.31°	1.23°	24.55°	1.25°	1.27°	1.12°

provides stepwise increasing orientation data with large value steps whereas the proposed filter calculates increasing orientation data narrow to the ground truth as can be seen in Fig. 3 (right) for $\omega = 3^\circ s^{-1}$. Table I summarizes the results for all test sets subdivided into the three categories of motion. The proposed filter can withstand with commercially available devices and is superior at slow dynamic motion regarding orientation estimation. Even at low angular rates of $3^\circ s^{-1}$ the proposed filter is still able to provide good estimation results with the same sensor data as the commercial algorithm.

Furthermore, Fig. 4 indicates, that the proposed algorithm is able to maintain orientation under magnetic disturbance. Typical results of orientation estimation with and without switching between MARG and IMU equations are presented. For fast dynamic motion at $16^\circ s^{-1}$ the disturbance is introduced 5 seconds after the start of the set and kept for an interval of 35 seconds. At $8^\circ s^{-1}$ the disturbance is introduced twice, at 15 seconds after start with a length of 20 seconds and at 40 seconds after start with a length of 15 seconds. Switching between MARG and IMU equations inside the GDFS enables to maintain orientation in static cases as well as for dynamic motion while magnetic disturbance is present. In contrast, the estimated yaw angle will experience large errors without the proposed switching.

However, in the case of magnetic disturbance lasting more than 35 seconds the orientation estimation will experience large errors in the heading estimate. This is due to the gyroscope bias and gyroscope resolution making an orientation estimation without large correction steps and growing errors impossible.

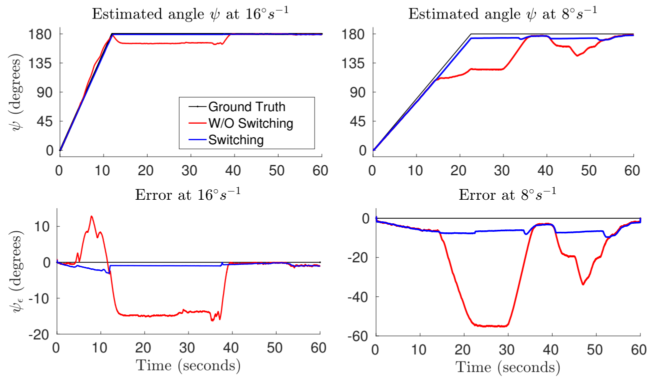

Fig. 4. Typical results for estimated angle ψ with (blue) and without (red) threshold-based switching (top) and corresponding error (bottom) under magnetic disturbance at fast ($16°s^{-1}$) and slow ($8°s^{-1}$) movements.

VI. Conclusion

Cascaded two-step Kalman filter approaches to estimate orientation of MARG-sensors and overcome short time magnetic disturbances are reported since 2014. These filters are based on large models and equation sets which significantly increase complexity and the need for optimal parameter tuning. The filter approach presented in this paper reduces the Kalman filter equations to the gyroscope only, therefore reducing model size and complexity to the well-known gyroscope dynamics. The calculation of the quaternion from gravity and magnetic field is based on the computationally efficient GDA. Under magnetic disturbance the proposed filter enables switching between MARG and IMU mode. This allows to maintain orientation and therefore increase robustness in environments which are exposed to short term magnetic disturbance. Separating the gradient descent quaternion from the gyroscope enables a motion based orientation estimation to be computed thus prioritizing the strengths and weaknesses of the different sensors and allow a continuous check for magnetic disturbance. Using a Kalman filter improves performance regarding gyroscope errors in phases of magnetic disturbance keeping drifts relatively low. In contrast to Madgwicks original approach, this filter does not need to be calibrated. Without valid magnetometer calibration, the filter will act as an IMU, switching to the equivalent subset of equations, whereas a valid magnetometer calibration will enhance the performance and precision of the orientation estimation.

VII. Future Work

Future applications on orientation estimation using MARG-sensors, e.g. controlling robots [4] [18], tracking posture [19] or monitoring movements [7] [10] require high precision and need to be robust to guarantee safety in environments compromised by magnetic disturbance. The proposed filter is a step to fulfil the robustness required in such tasks, by rejecting short-term magnetic disturbance and keeping the complexity low. Future work will focus on improvements of robust orientation estimation regarding magnetic disturbance and performance compared to other state of the art filter methods. Further increase in precision and reduction of errors, regarding the ability to overcome magnetic disturbance will be investigated by adding the magnetic field norm as another source of reference and combining it with the proposed filter.

References

[1] D. Kingston and R. Beard, "Real-time attitude and position estimation for small uavs using low-cost sensors," in *AIAA 3rd unmanned unlimited technical conference, Workshop and exhibit*, 2004, paper 2004–6488.

[2] G. Gupta, P. Barlow, and S. David, "Review of sensors and sensor integration for the control of a humanoid robot," in *IEEE Instrumentation and Measurement Technology Conference (I2MTC)*, 2011, pp. 1–5.

[3] P. Picerno, A. Cereatti, and A. Cappozzo, "Joint kinematics estimate using wearable inertial and magnetic sensing modules," *Gait & posture*, vol. 28, no. 4, pp. 588–595, 2008.

[4] N. Rudigkeit, M. Gebhard, and A. Gräser, "Towards a user-friendly ahrs-based human-machine interface for a semi-autonomous robot," in *2014 IEEE/RSJ International Conference on Intelligent Robots and Systems, Workshop on Assistive Robotics for Individuals with Disabilities: HRI Issues and Beyond*, 2014.

[5] W. De Vries, H. Veeger, C. Baten, and F. Van Der Helm, "Magnetic distortion in motion labs, implications for validating inertial magnetic sensors," *Gait & posture*, vol. 29, no. 4, pp. 535–541, 2009.

[6] D. Gebre-Egziabher, G. Elkaim, J. D. Powell, and B. Parkinson, "A non-linear, two-step estimation algorithm for calibrating solid-state strap-down magnetometers," in *8th International St. Petersburg Conference on Navigation Systems (IEEE/AIAA)*, 2001.

[7] X. Yun and E. R. Bachmann, "Design, implementation, and experimental results of a quaternion-based kalman filter for human body motion tracking," *IEEE transactions on Robotics*, vol. 22, no. 6, pp. 1216–1227, 2006.

[8] A. M. Sabatini, "Quaternion-based extended kalman filter for determining orientation by inertial and magnetic sensing," *IEEE Transactions on Biomedical Engineering*, vol. 53, no. 7, pp. 1346–1356, 2006.

[9] D. Roetenberg, H. Luinge, and P. Slycke, "Xsens mvn: full 6dof human motion tracking using miniature inertial sensors," *Xsens Motion Technologies BV, Tech. Rep*, 2009.

[10] S. O. Madgwick, A. J. Harrison, and R. Vaidyanathan, "Estimation of imu and marg orientation using a gradient descent algorithm," in *IEEE International Conference on Rehabilitation Robotics (ICORR)*, 2011, pp. 1–7.

[11] T. Pylvänäinen, "Automatic and adaptive calibration of 3d field sensors," *Applied Mathematical Modelling*, vol. 32, no. 4, pp. 575–587, 2008.

[12] D. Gebre-Egziabher, G. H. Elkaim, J. David Powell, and B. W. Parkinson, "Calibration of strapdown magnetometers in magnetic field domain," *Journal of Aerospace Engineering*, vol. 19, no. 2, pp. 87–102, 2006.

[13] D. Roetenberg, P. J. Slycke, and P. H. Veltink, "Ambulatory position and orientation tracking fusing magnetic and inertial sensing," *IEEE Transactions on Biomedical Engineering*, vol. 54, no. 5, pp. 883–890, 2007.

[14] S. Zihajehzadeh, D. Loh, M. Lee, R. Hoskinson, and E. Park, "A cascaded two-step kalman filter for estimation of human body segment orientation using mems-imu," in *Engineering in Medicine and Biology Society (EMBC), 2014 36th Annual International Conference of the IEEE*, 2014, pp. 6270–6273.

[15] L. Wang, Z. Zhang, and P. Sun, "Quaternion-based kalman filter for ahrs using an adaptive-step gradient descent algorithm," *International Journal of Advanced Robotic Systems*, vol. 12, no. 9, p. 131, 2015.

[16] R. G. Valenti, I. Dryanovski, and J. Xiao, "A linear kalman filter for marg orientation estimation using the algebraic quaternion algorithm," *IEEE Transactions on Instrumentation and Measurement*, vol. 65, no. 2, pp. 467–481, 2016.

[17] J. Wendel, *Integrierte Navigationssysteme: Sensordatenfusion, GPS und Inertiale Navigation*. Walter de Gruyter, 2011.

[18] T. L. Baldi, G. Spagnoletti, M. Dragusanu, and D. Prattichizzo, "Design of a wearable interface for lightweight robotic arm for people with mobility impairments," in *IEEE International Conference on Rehabilitation Robotics (ICORR)*, 2017, pp. 1567–1573.

[19] T. Mańkowski, J. Tomczyński, and P. Kaczmarek, "Cie-dataglove, a multi-imu system for hand posture tracking," in *International Conference Automation*. Springer, 2017, pp. 268–276.